MISSILE - FIRING CRITERIA FOR THE TACTICAL DOCTRINE PLANNER

WARREN TAYLOR

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

MISSILE-FIRING CRITERIA

FOR THE

TACTICAL DOCTRINE PLANNER

by

Warren Taylor

Lieutenant Commander, United States Navy

MISSILE-FIRING CRITERIA

FOR THE

TACTICAL DOCTRINE PLANNER

by

Warren Taylor

Lieutenant Commander, United States Navy

Submitted in partial fulfillment of

the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

1 9 5 8

MISSILE-FIRING CRITERIA

FOR THE

TACTICAL DOCTRINE PLANNER

by

Warren Taylor

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

from the

United States Naval Postgraduate School

ABSTRACT

This paper is addressed to tactical doctrine planners at all levels of responsibility in their service chains of command. It presents a non-technical discussion of certain missile-firing problems confronting the planner and offers criteria for the solution of these problems. The cumulative binomial distribution is used as a mathematical model of survival probability applicable in the event of atomic or nuclear warfare. The model becomes a measure of effectiveness for the planner to decide which is the better of several possible alternative firing doctrines (or ship formations, missile battery locations, etc.) he may have available. The model may be extended to handle the allocation of aircraft or missiles to enemy targets on a global basis. A second measure of effectiveness is developed for conventional-weapons warfare. The latter model strongly considers the high cost of present-day missiles and offers a basis for determining the number of missiles to be fired at any given target when the kill probability of the missile is known. The possibility of incorporating an observation period in the missile-firing cycle is analyzed and presented in the form of nomograms for decision purposes. It is concluded that the employment of such an observation period would be beneficial in connection with beam riding missiles under certain conditions of the input parameters and might even be feasible for homing missiles if a low, missile-operability factor exists.

PREFACE

The inception of the thoughts presented in this paper took place during the summer of 1957 while the author was temporarily assigned to the Operations Research Group of the Convair Missile Division of the General Dynamics Corporation at Pomona, California. Although the personnel of this group were largely concerned with a particular product at the time, the background discussions were of considerable help in formulating a general problem for this presentation. In this respect, the author is indebted to Messrs. Aubrey T. Gray and Richard E. Trueman of Convair for their suggestions and guidance concerning the art of missilery and the subject of firing doctrines. Mr. Trueman had studied the matter of survival probability [1] and introduced the subject and its possible use to the author. Part of this paper concerns itself with a similar model and its application from the tactical planner's viewpoint.

The material of Chapter II (considerations of Conventional-Weapons Warfare) was, in a sense, an experiment in the unknown in that it was originally begun with no particular goal in mind. The development, however, revealed mathematical substantiation of certain elements that were intuitively held to be true, and the conclusions very closely approximate results produced by other techniques involving considerably more labor.

The possibility of incorporating an observation period of some sort into the missile-firing cycle has been investi-

gated before with somewhat mixed conclusions [2], [3]. The material herein presented is a different approach to the problem and will lead to definite conclusions when viewed in the light of specific parameter values.

It may be noted in perusal that this paper contains few references to associated material. The reasons for this situation are twofold: a major portion of the literature of missilery is classified --- in order to avoid disclosure of such material, the author has pointedly not referred to classified material where it does parallel or substantiate this paper ---; search of relevant papers reveals a plethora of articles on design and reliability of future systems, but a dearth of information concerning how best to use these projected products as well as those now available. This latter situation would seem to indicate a fertile field for the analyst.

The rather extensive library at the Naval Postgraduate School contains but one thesis written on the subject of missile systems as defense units [4]. This article considers a measure of effectiveness (cost per kill or kills per dollar) with which this writer is somewhat at odds (see Chapter II and [7].). A recent thesis by Captain J.F. Tucker, USN, [5] concerns itself with a subject closely associated to that undertaken here --- the torpedo firing problem. The comparison of missile and torpedo firings is interesting but sufficiently apart that the mathematical models of analysis are necessarily different.

For their kind patience, help, and constructive criticism

on the organization and content of the paper, the author expresses his sincere appreciation to Professors W.P. Cunningham and Thomas E. Oberbeck of the Naval Postgraduate School faculty.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

MISSILE-FIRING CRITERIA

FOR THE

TACTICAL DOCTRINE PLANNER

* * * * * *

Warren Taylor

INTRODUCTION

The addition of new weapons and the improved performance of older weapons in the modern defense picture has created a need for the re-examination of firing doctrines as they have existed for many years. In particular, it is proposed to consider the usage of short-range missiles in the light of some of these developments. To discuss a specific system or specific true values of the parameters would, of necessity, classify the study for security reasons. It is intended, therefore, to use representative values where necessary for illustrative purposes to avoid such classification and thus to provide wider dissemination of the thesis. It is hoped that the material herein presented will provide food for thought and a basis for the determination of actual firing doctrines in specific installations by appropriate tactical doctrine planners.

In addressing the paper to "tactical doctrine planners", the author is writing to a large group of people who occupy various levels of authority in a service chain of command. The material to be presented is not only applicable at the field or shipboard level, but would also be of particular value to the staff planner who may issue directives concerning firing doctrines to subordinate units. The highest-level planners might well incorporate such material in their considerations when issuing official service publications, or when devising strategic or logistic plans for future operations.

To come more to the point, the problem at hand may be stated thusly: it appears that firing doctrines and procedures for missile installations are not as firmly established by directive from higher authority as are such doctrines for more conventional weapons. In some cases, individual units have been left to set up their own procedures in this respect on the basis of their current experiences or information from any available source. In time, this situation will surely change and firm procedures will be established at higher levels. During this interim period, however, someone must set forth doctrines for use in the event of war today --- this someone is a tactical planner who, today, may be occupying a relatively low-level position in his service organization. The author, therefore, addresses his work to this group of people as well as to the higher-level planners in the sincere hope that it can provide further insight to the problems of missile firing and aid in offering solutions for specific matters for whomever may be confronted with missile doctrine planning at the moment.

Today's firing doctrine must consider a multitude of problems heretofore unpresented. This investigation will discuss aspects of some of these major items. Doctrines will be divided into two categories at the beginning by considering special-weapons warfare and conventional-weapons warfare. The conventional-weapon discussion will then consider two aspects of the problem confronting a planner: the number of missiles to be fired and the incorporation of an observation period into the firing cycle.

It seems intuitive that the measure of effectiveness for a defense system might well be different under an attack which, if successful, will obliterate the defending forces or the installation they are protecting with almost a certainty versus that of an attack which, when successful, will damage the defending forces to a greater or lesser extent with an attached probability of something less than one. Again, the cost of the weapon of today, the missile, is considerably higher that the cannon balls of yesteryear. Defending forces are, to an extent, missile-limited from a cost and productive-capability viewpoint. Or, they may actually be tactically limited by the number of such pieces of hardware they have on hand in their magazines. Surely, this, too, must affect the measure of effectiveness.

Considerations such as the above led to the division of this paper into its present sub-heads. It is realized that the general problem could be approached from many directions. The division could be made by considering saturation attacks as compared with something less than this form, such as attacks of a wave form, stream, or by single snoopers. Each of these latter classifications should account for a complete spectrum of target altitudes and speeds. And then one would like to include the effects of a variable number of missiles per salvo and varying system time delays. Complete investigation of each of these cases, regardless of the initial division of the problem, would increase the complexity and content considerably and would seem to serve little purpose in a thesis of this type. Instead, effort will be made to cover

the scope of aspects such as these by treating certain subjects parametrically and others by illustration with realistic values while still avoiding reference to any known or planned missile or missile system.

The thoughts presented, then, will be a possible starting point for those interested in the field, a point from which, with specific inputs in mind, progress can be made in the formulation of more effective firing doctrines for today's military installations.

CHAPTER I

SPECIAL-WEAPONS WARFARE

A very simple measure of effectiveness describing systems designed to defend against an aircraft bearing atomic or nuclear weapons would be one which measures the effectiveness of the possible defense doctrines by the probability of survival of such an aircraft. If each trial (or missile firing) is assumed an independent event:

P_k = probability of kill of the aircraft in one shot (it is envisioned here that this quantity will be available to the planner essentially as a function of range)

P_s = probability of survival of the aircraft

m = number of missiles fired

Then, for the several cases:

(1) where the target is orbiting or circling the firing installation at a fixed range (a reconnaissance plane);

(2) several missiles are fired simultaneously at the same target; or

(3) the computation for survival is desired on an inbound target and the average P_k over the firing interval is considered (this assumption will produce a close approximation);

the following holds:

$$P_s = (1-P_k)^m$$

where P_k may also be thought of as the salvo kill probability and m as the number of salvos fired (see Appendix I for illustration).

Using this simple criterion of survival probability as a measure of effectiveness, the doctrine achieving the minimum probability P_s would then be measured as best. But such a measure, although considering the number of missiles fired (m), has not penalized the effectiveness for excessively large m. Obviously, if no penalty is paid for missiles launched, near perfection can be achieved by increasing this m to some large number --- with a P_k of only .4, ($m=$) 10 missiles could be fired and the desirable low measure of ($P_s=$) .006 would be achieved.

This simple measure would thus seem unsatisfactory since it does not offer the defense a true picture of performance unless the comparison of doctrines is made for a specific value of m. Yet the defense does not want to consider itself missile-limited by making such a specification of m in that it is really willing to fire all of the available weapons if necessary to deter this enemy effort (---the alternative is certain destruction by this enemy). The latter thought might suggest considering the effort the enemy must put forth to evade successfully the defense system, that is, the number of aircraft (N) the enemy must mount to give P_n probability that n or more aircraft will survive the defense.

Evaluation of the required number of aircraft or raid size (N) is dependent on both missile kill probability (P_k) and the system saturation level (N_s), the latter being the maximum number of aircraft that can be engaged at any one instant using a particular doctrine. Until the saturation level is reached, the probability of any given aircraft surviving the defense is dependent only on P_k. This probability

(P_k) itself depends on the probability of detection and identification, the reliabilities of the missile and ground system as well as their operability factors, the tracking reliability (for beam riders), the lethality of the warhead, the reliability of the fuze, type and size of the target, its course and speed, and numerous other variables. The assumption is made, however, that an overall P_k is known and the complete range ($0 \leq P_k \leq 1$) of this quantity will be investigated.

If it is desired to construct a mathematical model for the probability P_n that the number of aircraft surviving a raid of N aircraft will equal or exceed n, given that the probability of kill of the missiles being fired at each of the raiding planes is P_k, the following procedure can be employed:

the probability of any one of the N planes surviving

$= (1-P_k)^1$

the probability of all of the remaining (N-1) planes being shot down $= P_k^{N-1}$

the probability of these events occuring together

$= P_k^{N-1}(1-P_k)^1$

the selection of the one plane from N planes

$= {}^NC_1$

Therefore, for survival of

1 plane, $P_1 = {}^NC_1 \; P_k^{N-1}(1-P_k)^1$

2 planes, $P_2 = {}^NC_2 \; P_k^{N-2}(1-P_k)^2$

--- , ---

x planes, $P_x = {}^NC_x \; P_k^{N-x}(1-P_k)^x$.

For the probability P_n of the survival of n planes, or (n+1) planes, or (n+2) planes, and so on (that is, to "equal or exceed n") out to the total N planes, the result is

$$P_n = {}^N C_n P_k^{N-n}(1-P_k)^n + {}^N C_{n+1} P_k^{N-(n+1)}(1-P_k)^{n+1} + \cdots + {}^N C_N P_k^{N-N}(1-P_k)^N$$

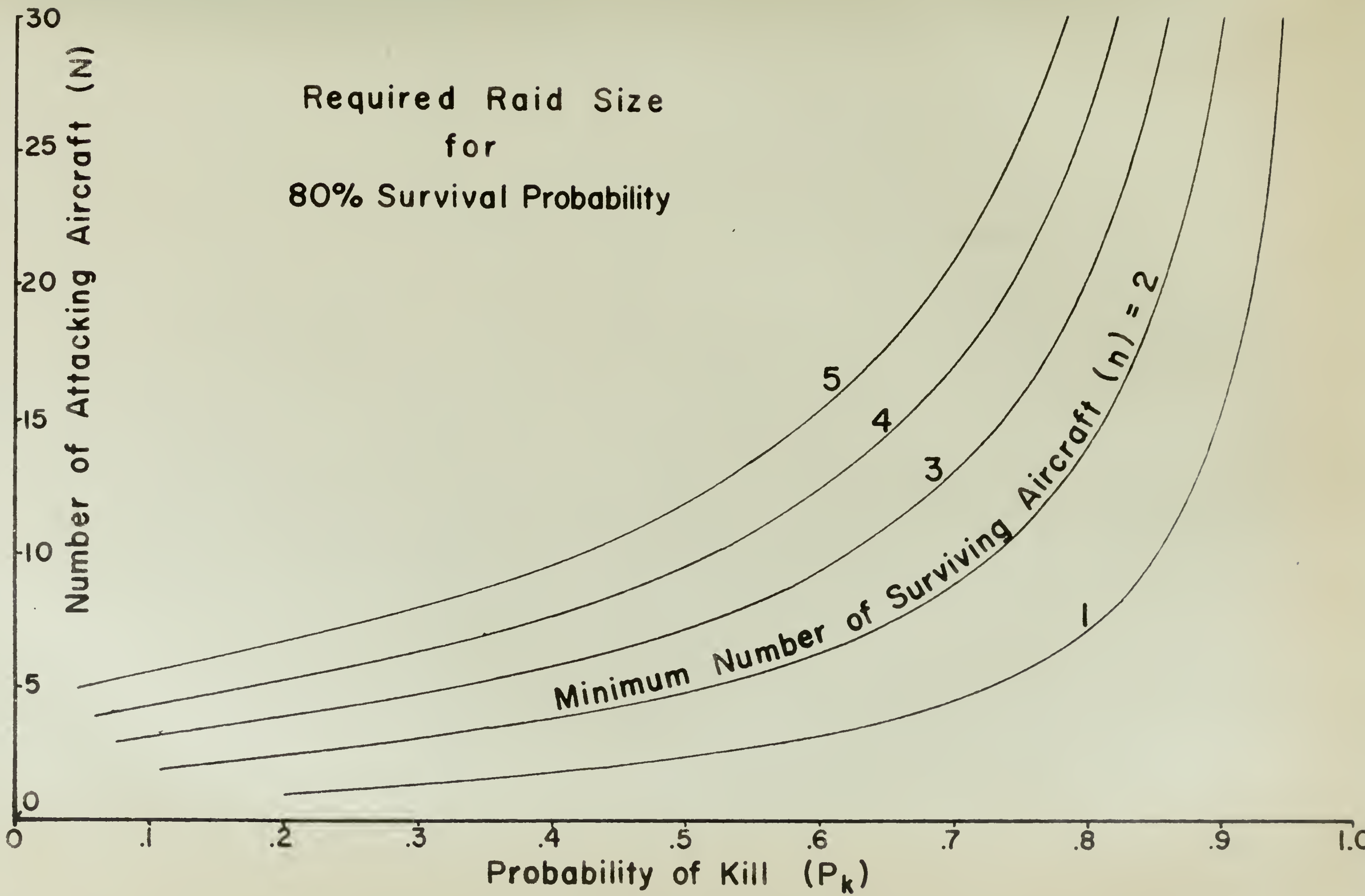
or

$$P_n = \sum_{x=n}^{N} {}^N C_x P_k^{N-x}(1-P_k)^x$$

It is seen that this probability is expressed conveniently by the Cumulative Binomial Distribution for which there are tables available [6] for variable N, x, and P_k to give P_n.

Assuming, for instance, that the enemy is willing to pay the attrition price of a .80 probability P_n that n or more of their aircraft will survive, the function developed above can be plotted for a spectrum of N and P_k. Figure 1 presents such a plot. The curves show the interrelationship of N and P_k. It is seen that, for a given raid size (N), as the kill probability (P_k) of the defending installation increases, the enemy has less and less favorable chances of having n (or more) survivors get through the defense. Viewed differently, for a constant P_k, the number of survivors increases as the raid size increases.

In order to utilize this mathematical model as a measure of effectiveness the planner should first determine the saturation level (N_s) of a doctrine he desires to examine. This level is a function of the inherent system delays (reload, target assignment, tracking time, computor solution, launcher synchronization, target parameters such as speed, altitude, spacing, etc.) and can be computed from modal (most likely)



Required Raid Size
for
80% Survival Probability
Number of Attacking Aircraft (N)
30
25
20
15
10
5
0
Minimum Number of Surviving Aircraft (n) = 2
1
3
4
5
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
Probability of Kill (P k)

Figure 1

practice times plus representative values for the target parameters. Making a similar computation for a second doctrine with which it is desired to compare the first, it is found that the relative effectiveness of the two can quickly be measured.

As an example, consider two doctrines A and B where A might be a doctrine which proposes firing two missiles at each target and B proposes firing single missiles instead. With two director systems and their associated launchers available and using the average P_k over the (inbound) firing distance, it might well be determined that A can only fire on half as many targets as B (i.e. saturation level of A is half that of B) due to a high launcher reload time. Assuming the saturation level of B is, say, 8 targets, and that the single-missile kill probability P_k is .4, the parameters become:

A: $N_s = 4$, $P_k = .64$

B: $N_s = 8$, $P_k = .4$

<u>Comparison of Two Firing Doctrines</u>

Number of attacking aircraft	.80 probability of n or more aircraft surviving the defense	
N	n_A	n_B
1	0^*	0^*
2	0^*	1^*
3	0^*	1^*
4	1^*	2^*
5	2 $(5-4+1^*)$	2^*
6	3 $(6-4+1^*)$	2^*
7	4 $(7-4+1^*)$	3^*
8	6 $(8-4+2^*)$	4^*

9	7 (9-4+2*)	5 (9-8+4*)
10	8 (10-4+2*)	7 (10-8+5*)

* From Figure 1.

(For N=5, the comparison shows that doctrine A allows 1 plane to get through with a 100% certainty (since saturation level is 4) and 1 or more with a probability of .80, hence n_A=2; B allows for 2 or more with a probability of .80, hence n_B=2.)

The measure would seem to indicate here that for attacking aircraft numbering 4 or less, it would be better to employ doctrine A; for numbers greater than 4, doctrine B is more effective. (See Appendix II for a further discussion of this matter.) The Gunnery Officer can, of course, make such a determination (of raid size, and knowing the approximate open-fire and cease-fire ranges and hence P_k) as the raid approaches from a distance, and issue the appropriate instructions based on his prior comparison by a method such as this.

The example used for illustrative purposes considered values of P_k and N_s introduced by a variation in the number of missiles per salvo. It should be realized that the method of evaluation is general and will handle values of P_k and N_s introduced by a variation of any sort. In a shore installation, for instance, different parameter values could very well be introduced by different battery locations; the method could be used to evaluate the effectiveness of different formations involving several defending ships in a similar manner. Appendix II indicates further uses for this particular measure of effectiveness from an even broader viewpoint.

In a way, this scheme of evaluation avoids direct consideration of the number of missiles fired. It throws the burden of "proof of effectiveness" on the enemy (although, of course, he remains unaware of this fact). It says to him, "How many planes are you willing to put in the air to insure that you will get 1 (or 2, 3, etc.) through to the target?" And the doctrine planner says, "I will measure the effectiveness of my doctrine by the amount of effort you have to put forth to accomplish your purpose." In a global war, the enemy obviously might not be able to pay continuously the price to defeat such maximized schemes and the planner has achieved his desired effectiveness. It was seen that the method applies to saturation as well as non-saturation attacks and handles single snooper raids, stream attacks, or wave attacks depending on the manner of computation of N_s. Actually, computation for the worst form of attack, the wave form, will produce a lower N_s than for a stream attack, but, in any case, a tabulation such as that shown above will offer a quick choice as to the best of several possible doctrines under the attack conditions. It is suggested that such tabulations be made for representative target altitudes and speeds based on the fixed, known parameters of the particular defending installation, and then be reduced to specific procedures to be followed for enemy attacks with special weapons.

CHAPTER II

CONVENTIONAL-WEAPONS WARFARE

In considering the formulation of firing doctrines applicable to conventional-weapons warfare, it should be kept in mind that the expected damages to be sustained by the defending installation are considerably less than under conditions of atomic or nuclear warfare. The matter of acceptability, or the cost of carrying out a decision to a successful conclusion, assumes a greater importance. The reader must now divorce himself from further consideration of the special-weapons problem and its conclusions and consider missile firing in the light of this element of cost. A few additional thoughts on such matters seem appropriate.

Basically, the conventional form of warfare takes on the aspect of some sort of a (cruel) game --- the defense has certain resources, the attacking forces have resources. How much does the defense want to pay, or can they afford to pay, to inflict damage on the enemy, remembering that if the defense fails to harm the enemy target at all, the payload the enemy carries does not have that certainty of destroying the defended installation that must be assumed in consideration of special-weapons warfare? If the defense knew with a good degree of certainty, for example, that the attackers would drop a bomb which would cause no injuries and only cause minor shrapnel damage to a small portion of the defense installation, would they be willing to launch, say, six expensive missiles at the attackers? Four? Two? Any?

The defending forces do not know this information "with a good degree of certainty", but probability tells them that this is what they might "expect" in the way of damage from, say, a very high-flying, small-payload bomber. And, at the same time, it tells the defense that they have a poor chance of hitting this high-flying enemy plane at an extreme range an altitude (possibly even with the six missiles!), only a slightly better chance as the enemy target closes, and not too good a kill probability at the enemy's bomb-release line. How many missiles should be launched, if any? Would the many-thousand-dollar effort be worth the expenditure? Is it acceptable? This chapter will concern itself with such matters.

It is seen that a measure of effectiveness designed to cover this new set of conditions might well have different elements than that developed in the first chapter. It might also be somewhat difficult to define clearly. Strategic measures might deal with the tasks assigned to the defense system. If the missile system were quite capable of making targets fly higher over the defended area (through enemy fear of the missile-system capability) and thus fall prey to a friendly combat air patrol, or if it denied the use of slow reconnaissance planes to the enemy, it might well be an "effective" system for the strategic planner. But the field commander, commanding officer of a ship, or tactical planner is more vitally concerned with the actual use of the system than its general employment. Once it has been strategically designed, is it tactically most effective within the limits imposed? It must therefore be recognized that there may be

numerous "measures of effectiveness" for the conventional-weapons problem. The author will endeavor here to indicate the thought process in developing one such measure which, as will be seen, can be used to indicate the number of missiles to launch at a target for any given value of P_k.

One might feel that any measure of effectiveness should consider the number of planes or targets which penetrate the defense system. In this case, the defense would like to shoot missiles only at those targets that it knows will penetrate, hence some concept of conservation of missiles for the higher-P_k targets must be introduced in that these higher-P_k targets are presumably closer to their desired destination and hence have a better probability of penetration. It should be remembered, however, that to penalize a doctrine by low-rating it in cases where targets did, in fact, penetrate but could not be taken under fire for lack of missiles on hand to launch would be a bit unfair. And this would surely occur if penetration itself were the sole criterion.

A counting of just bursts themselves (which might be induced by a method advocating the averaging of kill probability over an area) does not really give proper weighting to individual bursts. Surely some of these bursts have a greater kill probability attached to them than others. The value of the present-day missile alone would seem to dictate not considering it in the same class as the conventional projectile and fuze of several years ago. It is felt that bursts themselves, or fire power, as a measure of effectiveness would only be valid (if at all) in comparing two doctrines differing in this

quantity alone.

Kills per missile or kills per dollar are similarly unsatisfactory since they over-rate a doctrine that fires but a few missiles at only "sure bets". Walsh discusses this matter at some length in his "Inadequacy of Cost per 'Kill' as Measure of Effectiveness" in a recent issue of The Journal of the Operations Research Society of America. [7]

When one attempts to inject "value of a target" into the problem, the tactical planner is left with a guessing game as to his true effectiveness, for who can say what the mission or pay-load of any given enemy aircraft is? "Value of the defense area" similarly opens the discussion to many issues beyond the realm of clear mathematical analysis with the techniques presently available.

It is felt that "proper utilization of missiles on hand" should have some bearing on the measure --- if the defending installation has only one missile to launch and successfully obtains a kill with this device it is surely 100% effective within the limitations imposed despite the possibility that more targets may have been available. Consideration of this factor (i.e. proper utilization) also introduces the dollar element into the picture which has a natural importance. On the other hand, if the defense had two missiles available, launched one and obtained a kill, yet failed to engage a second available target, there should be some penalty paid in the judging of performance, since it is evident that complete "proper utilization" was not employed.

It thus appears that the important ingredients are "kills

obtained", "utilization of missiles available", "number of kills available", and "missiles actually fired". These elements may be combined by multiplying the kills actually obtained per kill that was available ("within envelope" or "available missilewise") times the kills obtained per missile fired. Then, a doctrine which fails to kill all of its "available" targets will be penalized accordingly on a relative scale, and one which fires more than the necessary quantity of missiles will receive a similar lower rating. The function thus becomes:

$$\text{M.E.} = \frac{E^2}{mE_a}$$

where

E = expected kills

m = missiles fired

E_a = maximum single-shot expectation of kill (given that the target is "within the envelope" and there is a "missile available") per plane times the number of planes.

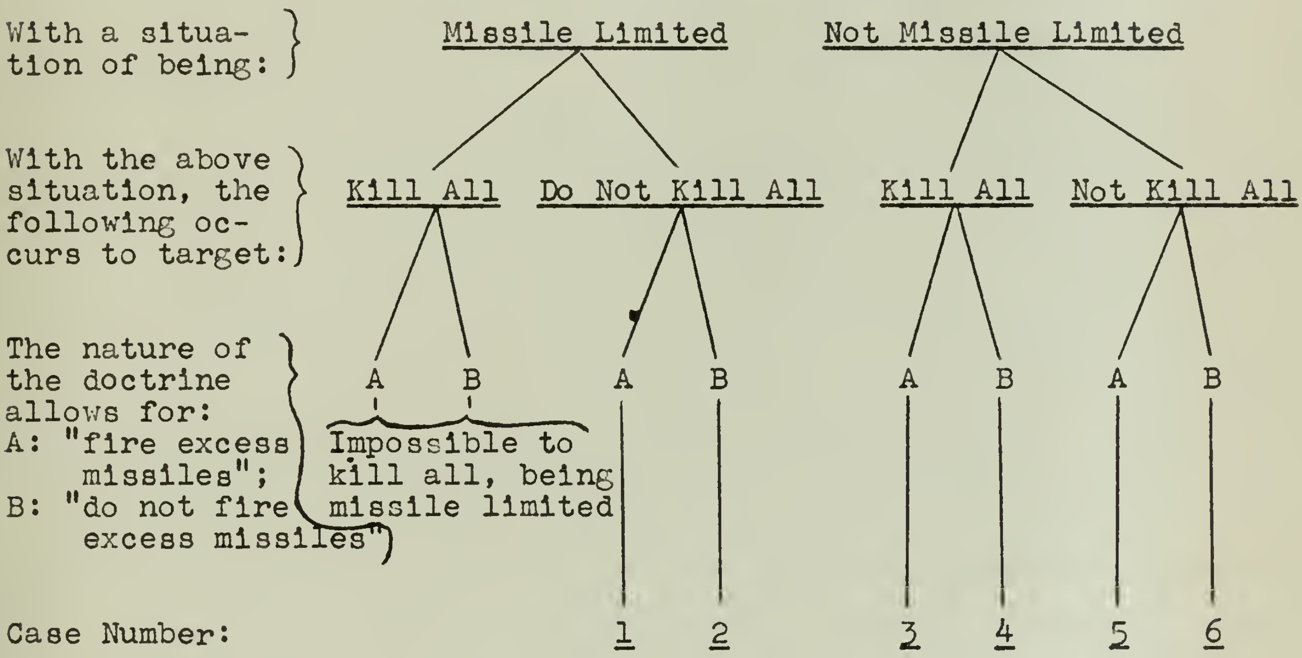
It is seen that the units are:

$$\text{M.E.} = \frac{(\text{kills})^2}{(\text{missiles})\left(\frac{\text{kills}}{\text{plane}}\right)(\text{planes})} = \frac{\text{kills}}{\text{missile}}$$

It should be noted that the eventual units are similar to those of a measure previously cast aside. These "kills per missile", however, are somewhat removed from the first proposal by the same name as will be seen in the below illustrations. The measure itself is an efficiency rating in that it represents an output (plane kills) divided by an input (missiles fired). Although the measure contains several familiar elements,

some of which are frequently considered effectiveness measures themselves, particular care need be exercised should it be used to compare, for instance, the capabilities of different missiles or missile systems.

In order to examine this measure and see how representative examples compare the following possible cases will be considered:



Assuming $P_k = 1.0$ (for facility of presentation and understanding of the illustration) and examining the above six cases in tabular form:

Sample Performance Ratings

Case Number	No. of a/c in raid	No. of miss./ salvo	No. of miss. fired	No. of kills obtain-ed	No. of miss. in stock	M.E. = ___	
1	20	2	10	5	10	$\frac{(5)^2}{(10)(10)}$	.25
2	20	1	10	10	10	$\frac{(10)^2}{(10)(10)}$	1.00
3	20	2	40	20	40	$\frac{(20)^2}{(40)(20)}$	.50
4	20	1	20	20	40	$\frac{(20)^2}{(20)(20)}$	1.00
5	20	2	20	10	40	$\frac{(10)^2}{(20)(20)}$	.25
6	20	1	10	10	40	$\frac{(10)^2}{(10)(20)}$	.50

<u>Case 1</u> illustrates how the measure penalizes for firing excessive missiles (2 per target) and in addition for not producing results commensurate with the reasonable expectation (single-shot, expected kills = 10, actual kills obtained = 5).

<u>Case 2</u> rates the performance 100% effective for getting 10 planes for 10 missiles despite the fact that 20 targets were available since it was stated that only 10 missiles were available (or 10 "kills" within range as an alternative usage of this device).

<u>Case 3</u> shows the result of firing too many missiles but still killing all available targets.

<u>Case 4</u> presents 20 kills for 20 shots in a raid of 20, therefore 100% effective.

Case 5 demonstrates a measure for a system which shoots too many missiles and does not even use these effectively. This case compares no better than Case 1 which shot down only half as many targets but was missile limited. Presumably, if system 1 were not so limited, it would have doubled up and shot 10 planes for 20 missiles, being no more effective as a system (i.e. not having received an additional penalty for this condition).

Case 6 shows the result of taking only half the available targets under fire and killing all of these --- 50% effective. This compares with Case 3 which illustrated missile waste.

Of course, this example was synthetic and almost trite, for one can never expect $P_k = 1.00$. The next step is to examine a practical problem now that the terms in the formula and their proper interpretation are more familiar.

Effectiveness Ratings

		Number of shots		
		1	2	3
P_k	.7	.7	.5915	.4508
	.5	.5	.5625	.5104
	.3	.3	.4335	.4796

(Note: P_k here does not include ground system operability)

The computation of the entries here proceeds as follows:

for the top right cell,

$$\frac{[.7 + .3(.7) + .09(.7)]^2}{3(.7)} = .4508$$

The table above clearly indicates that, as reliability

(or kill probability) increases, fewer missiles should be fired since the effectiveness rating drops off sharply after the first shot for these higher-P_k values. This would indicate the presence of some Law of Diminishing Return. Such indications are also presented by the probability of kill itself when viewed on a percentage basis. For one shot $P_{k_1} = .7$; for the addition of a second shot $P_{k_2} = .91$, or an increase in the original probability of only .21 (30% increase). This same missile, if fired with an original $P_k = .3$ would produce a second shot increase of the same .21 which is now a 70% increase in the original (.3) kill probability! The conclusions to be drawn from both the cumulative kill probability and the effectiveness rating are that <u>if it is considered worth while to shoot one missile with $P_k = .3$, it is worth while to shoot 2, and, as a matter of additional fact</u> (as will shortly be seen)<u>, 3, or possibly even 4!</u>

Reflection on the material developed to this point reveals that a rather awkward tool for practical use has been produced. The indications are clear that a "picture" is needed at this point to aid in the thinking process. The Effectiveness Rating vs Rounds Fired for various values of P_k is plotted and included here as Figure 2.

From the peaks of the curves in Figure 2, one obtains the curve of Rounds Fired for Maximum Effectiveness vs P_k shown here as Figure 3. The Percent Increase in Kill Probability (vs Rounds Fired After First Round as the abscissa) created by these (additional) rounds is also presented for

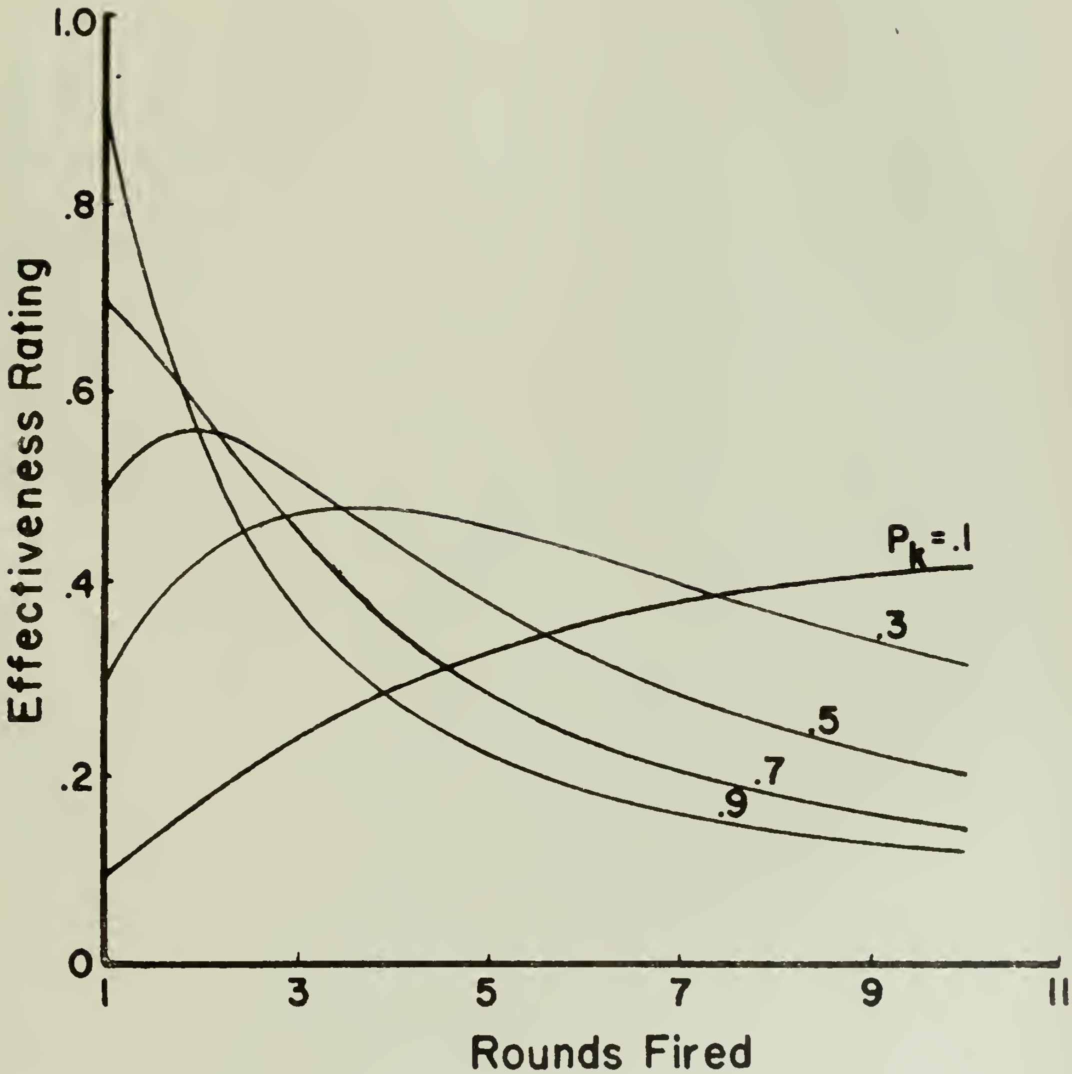

1.0
.8
.6
.4
.2
0
Effectiveness Rating
1
3
5
7
9
11
Rounds Fired
P_k = .1
.3
.5
.7
.9

Figure 2

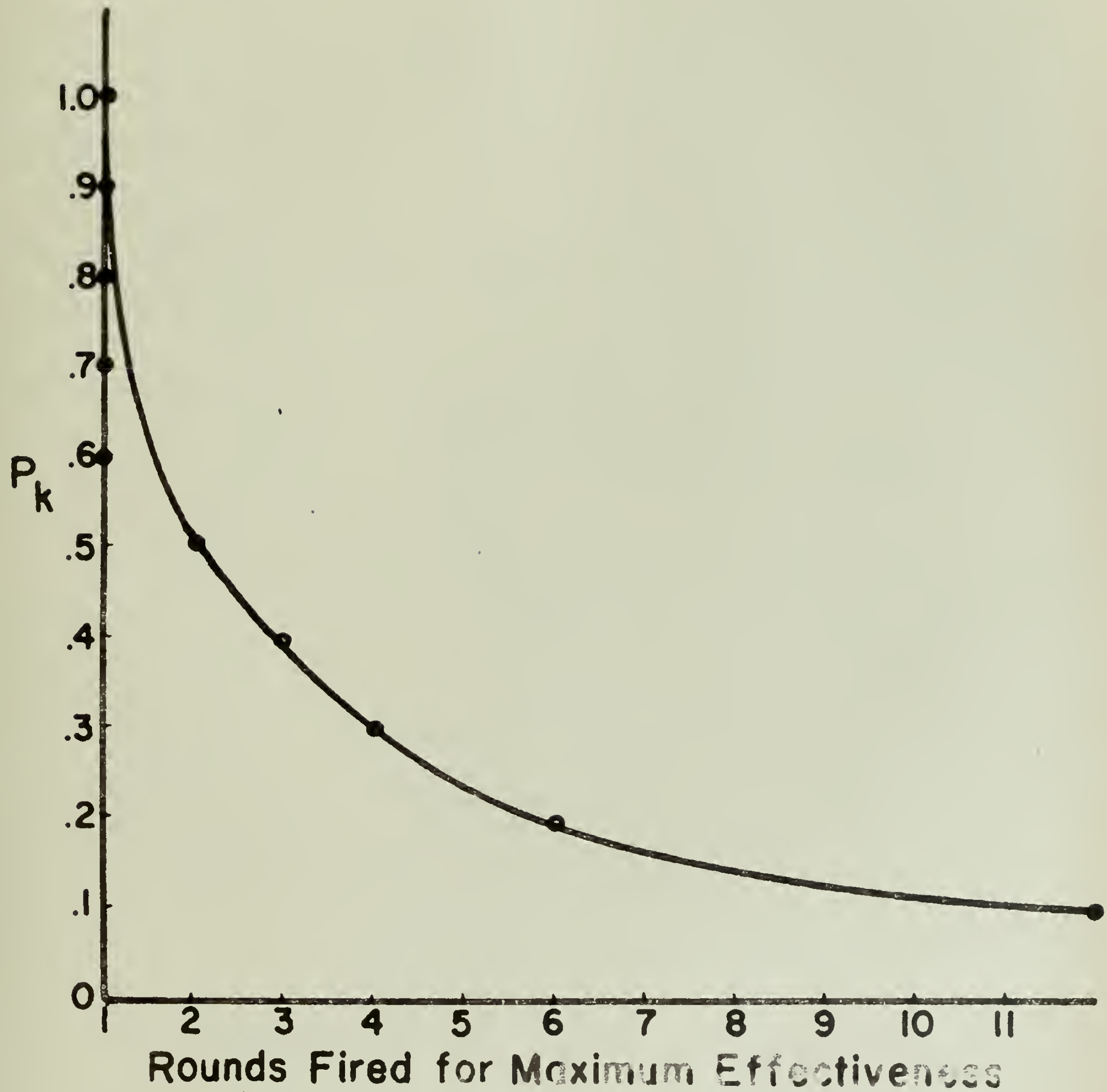
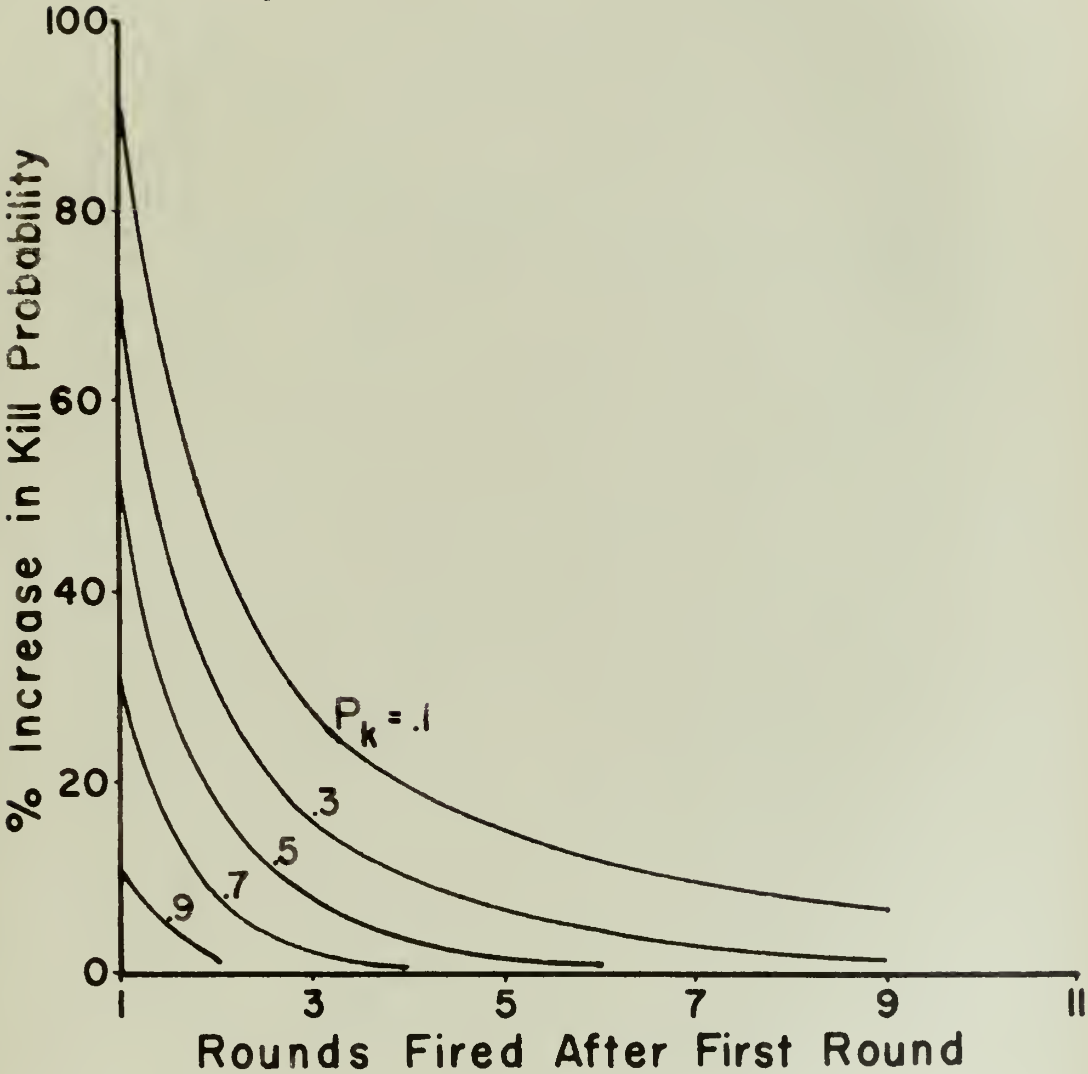

1.0
.9
.8
.7
.6
P_k
.5
.4
.3
.2
.1
0
1
2
3
4
5
6
7
8
9
10
11
Rounds Fired for Maximum Effectiveness

Figure 3

convenience (Figure 4) as is the Cumulative Kill Probability (vs Rounds Fired) (Figure 5). The latter plot indicates where the Maximum Effectiveness Curve lies in relation to the Cumulative Kill Probability. Although the discrete nature of rounds fired makes it somewhat difficult to present an accurate and complete picture of all possible cases, it is seen from an examination of Figure 5 that the rating scheme says that a cumulative kill probability of (about) .7 (or better, if P_k itself is greater than .7 to begin with) is near optimum. When effort is made to get a higher cumulative kill probability than this value, more missiles are expended than really should be (that is, input is too great for the output attained). The Law of Diminishing Return is affecting the results in this region.

This particular measure of effectiveness might be rather difficult to work with as a tactical aid as has been implied above, but it can aid the doctrine planner in his thinking and computations. For instance, should the doctrine call for two-missile salvos or single-missile salvos? The measure indicates that the answer is variable depending on P_k: if the pick-up is close in (and P_k is high), shoot singles (see Figure 3). The extra missile is costly to shoot and increases the cumulative kill probability only by a small percentage (see Figure 4). On the other hand, if the doctrine being planned is to cover wave attacks and early shots <u>must</u> be fired (low P_k) to enable the firing of <u>some</u> shots on all of the targets (or as many as possible), the number per salvo should be increased accordingly within the capabilities



100
80
60
40
20
0
% Increase in Kill Probability
$P_k = .1$
.3
.5
.7
.9
1
3
5
7
9
11
Rounds Fired After First Round

Figure 4

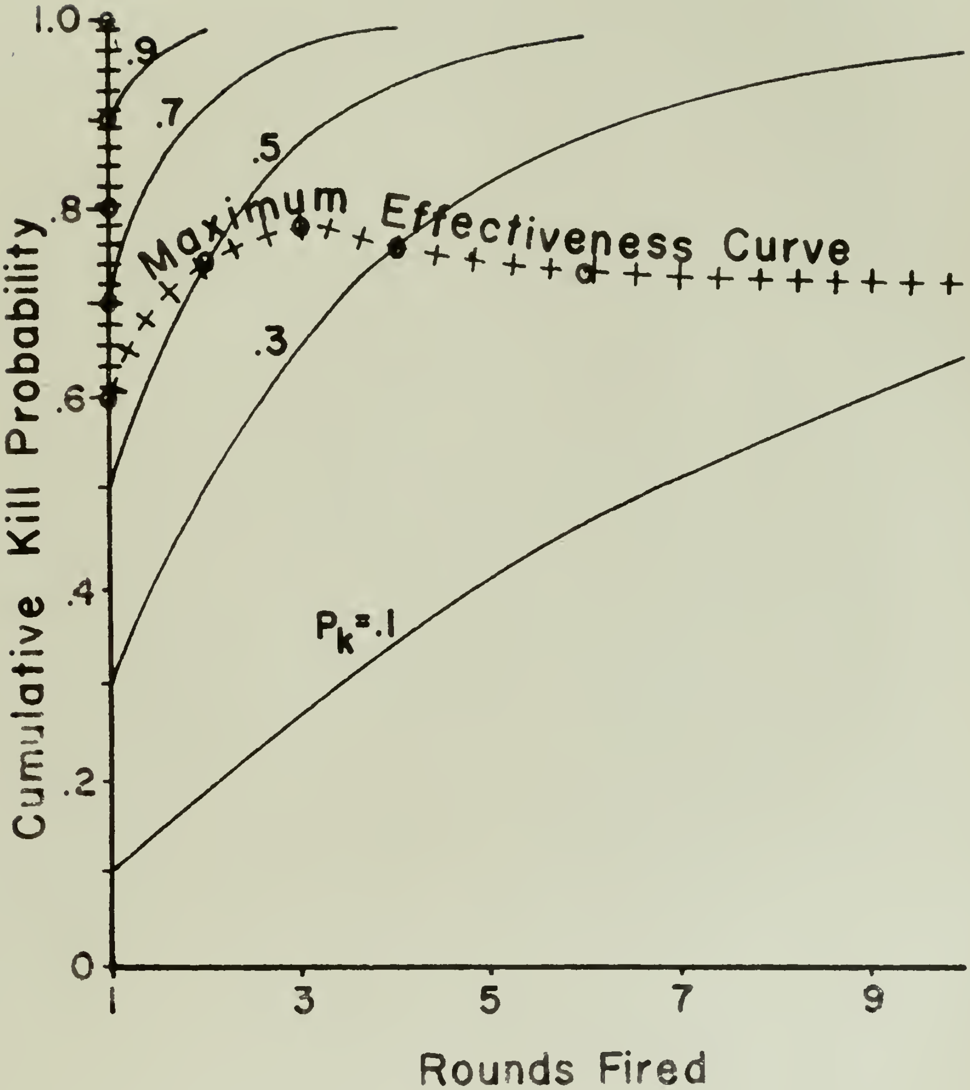

1.0
.9
.7
.5
.8
Maximum Effectiveness Curve
.6
.3
.4
P_k=.1
.2
0
1
3
5
7
9
Cumulative Kill Probability
Rounds Fired

Figure 5

of the system. Everything considered, a doctrine designed against single plane attacks or stream attacks (at sufficient interval) might well be to hold fire until the salvo P_k is about .65 or .7 (from the Effectiveness Curve of Figure 5).

A consideration of the problem confronting the planner at this exact point in the preparation of a good firing doctrine introduces the next section of this paper.

CHAPTER III

FIRE AND OBSERVE VERSUS FIRE AND SWITCH

Having provided for the firing of a salvo (single-missile or otherwise) determined in the manner proposed by the previous chapter (or by any other suitable criterion), the doctrine planner might well now wonder whether he should provide for the immediate slewing on to a second target and the preparation to fire at it, or whether he should incorporate an observation period in the firing cycle to see if his first salvo was successful in its purpose. The following terms and symbols are defined for a further study of this matter:

<u>Fire and Observe</u> --- a doctrine utilizing observation by radar or boresight telescope of the target after salvo burst for time t_o to note the presence or absence of a kill. If a kill is observed, a new target is engaged; if a kill is not observed within t_o, a second salvo is launched against the same target and the effect is again observed.

<u>Fire and Switch</u> --- a doctrine requiring each salvo to be launched against a different target (unless there are no different or unengaged targets in which case a previously-engaged target not yet destroyed may be re-engaged). For homing missiles, this infers transfer to a second target as soon as the first target is fired on; for beam riders, the switching process would occur immediately after a burst (not premature, however) on the first target.

<u>A Kill</u> --- target is observed to explode immediately upon

burst of missile. (In the discussion that follows, reference will be made to "missile kill probability", "missile bursts", and so on. "Salvo" can replace "missile" where desired if "salvo kill probability" is utilized in place of "missile kill probability" and a suitable counting of the numbers of missiles per salvo is employed where necessary.)

<u>B Kill</u> --- target is observed to commence falling within a few seconds after burst.

<u>C Kill</u> --- target will fall within a longer period after burst, say five minutes.

<u>D Kill</u> --- mission of the target has been thwarted (bomb-bay doors jammed, plane discontinues attack, etc.).

P --- eventual state of missile, where $P=1.0$ for success (kill obtained); $P=0.0$ for failure (kill not obtained).

t_b --- time from launch to burst.

t_f --- missile mean failure time (where failure occurs after launch and before burst).

t_l --- time to reload launcher.

t_o --- average time after burst for observing a kill.

t_s --- time required to switch to a different target (Note: this quantity will be measured from "leave the present target" to "fire at a different target".).

p_o --- probability of observing an actual kill within time t_o.

g_o --- ground system operability. It is considered that the ground system might be represented by the following bar diagram where the whole bar represents the total number

of trials of the event of ground system operability:

|//////S//////| U |

The unshaded area (U) includes all those times the ground system fails to operate when the appropriate switches are thrown. g_o then equals $\frac{S}{S+U}$.

g_r --- ground system reliability. Out of the segment S of the above bar, part of the time the operation will be reliable (R) and part unreliable (N) as follows:

$$\underbrace{|///R///|/N/|}_{S}$$

Not reliable times of operation would include periods when the equipment is out of tolerance. g_r then equals $\frac{R}{R+N} = \frac{R}{S}$.

m_o --- missile operability. Similarly defined.

m_r --- missile reliability. Similarly defined.

m --- number of missiles fired.

p --- kill probability of an operable missile (which is now the kill probability as a function of miss distance, the latter being a function of g_r and m_r).

T_i --- expected total engagement time for an i^{th} doctrine (three will be considered, hence i = 1,2,3).

P_i --- expected engagement kill probability for an i^{th} doctrine.

M_i --- expected mortality rate (kills/second) for an i^{th} doctrine, $= \frac{P_i}{T_i}$. M may then be thought of as the expected number of successes or kills to occur divided by the expected total time to achieve these successes. M will be used as the measure of effectiveness in the discussion

that follows. It has been selected in particular in that it not only considers the kills brought about by the use of a doctrine but also introduces the time required to produce these kills into a rate concept. The latter device is desirable since the effect of adding a time increment (t_o) in the firing cycle is being investigated. A high value of M, as seen from above, will be a desirable feature of any particular doctrine; if $M_i > M_j$, doctrine i will be considered better than doctrine j.

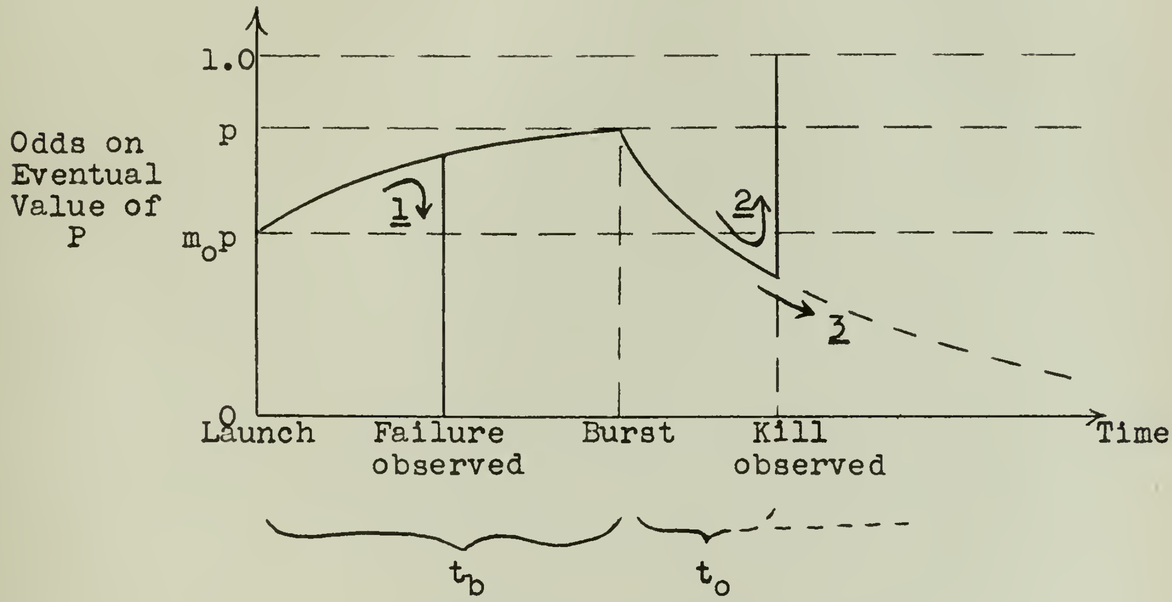
Assumption: firing time itself is negligible.

As the various types of kills defined above are considered, a planner might think he would like mostly type A's, then B's, C's, and D's in that order. For actual purposes, however, B kills or C kills (or possibly even D kills) are just as satisfactory as A kills. The synthetic device of typing kills is introduced by a lack of intelligence concerning the damage (if any) that has been done to the target, or a state of ignorance of the eventual value of P. Although the planner would like to be certain immediately of the event by observing an A kill ($t_o = 0$), he might be willing to wait for a type B before firing again ($t_o > 0$) to avoid the waste of "over kills" (should he decide to shoot at the same target again only to observe the target fall as a result of first missile damage after having already fired the second and it has not yet reached the target). It seems possible that he could even afford timewise to wait for some C kills ($t_o \gg 0$). In practice he would probably want to fire again rather than assume that a D kill has occurred ($t_o \ggg 0$).

Such thoughts have been entertained since the arrival of high-cost and high-kill-probability devices in the weapons field. In view of the fact that a fire-and-observe doctrine ($t_o > 0$) by its inherent nature generates a high probability of kill for each target engaged (since the doctrine assumes missiles are fired at any engaged target until a kill is observed) and an economy of missiles (since the "over kills" noted above are largely eliminated), but will increase the total time devoted to each target, it would seem appropriate to investigate the effect of this observation period on the mortality rate.

In considering the several quantities defined above, it should be noted that the quantity m_o might not be as clearly separative as g_o in that some missiles will be inoperable on the launcher and others, whether they be beam riders or homers, will only develop to be inoperable while they are in the air. The former group would equally affect any firing doctrine in that they must be replaced before firing can occur and such time of replacement must be added to the total time required for a cycle of any doctrine when it, in fact, occurs. This group will not be considered here. But the latter group can be identified at or before burst, and such identification together with the true value of m_o could well affect the feasibility of fire and observe versus fire and switch. Proper identification of this group would enable higher engagement kill probabilities and missile economy without paying the full penalty of a decreased rate of fire. Thus, before observing target kills, operability of the missile in the air

should first be observed. If the odds an observer would be willing to quote on the eventual state of P are examined as a function of time, the presentation would look something like the following diagram (Note: here, odds of "1.0" might mean the observer would be willing to offer 100 to 0 on missile success -- i.e. he is sure of the eventual state of P; "0.6" could signify 60 to 40 offer on success; etc.):



Odds on Missile Success versus Time

Path 1 shows odds on P = $m_o p$ while missile is on the launcher. As time increases after the launch, the odds increase as the missile continues to fly properly (capture occurs for beam riders, etc.). A failure occurs and the missile does not reach the target or burst does not occur. Thus, P drops to 0 and the odds the observer offers are 0 to 100 on success --- i.e. he is certain of failure and makes "no offer".

Path 2 starts as above, but continues on to a burst. The

observer now knows the missile was completely operable and the kill is now dependent only on the kill probability (p) of the warhead (early or late burst, vulnerability of target, etc.). As time increases, the odds begin to fall (due to a state of ignorance, the observer begins to lack confidence in P being 1.0 since he does not see the target fall), but P assumes a value of 1.0 as a kill is observed and the odds jump to 100 to 0.

For path 3, as time increases, the odds continue to drop off as, successively, an A kill is not observed, a B kill fails to occur, and so on. The odds would drop to 0 to 100 when/if the observer ever received intelligence to the effect that the mission of the enemy plane was, in fact, accomplished (bomb dropped, pictures taken, etc.), that is, he determined $P = 0$.

Consider now the three doctrines following where the time cycle begins at "fire" and ends at "ready to fire" a second shot:

Doctrine 1: fire and switch for a homing missile

$$P_1 = m_o p$$

$$T1 = t_s$$

$$M_1 = \frac{m_o p}{t_s}$$

Doctrine 2: fire and switch for a beam-riding missile

$$P_2 = p$$

$$T_2 = t_f\left[(1-m_o) + (1-m_o)^2 + --- + (1-m_o)^m\right] + t_b + t_s$$

$$\text{for} \begin{cases} t_1 < t_f, \text{ with a single launching rack available; or} \\ \text{a standby launcher available; or} \\ \text{more than one rack per launcher.} \end{cases}$$

Since the term $\left[(1-m_o) + (1-m_o)^2 + --- + (1-m_o)^m\right]$ is a geometric progression, the sum of the terms approaches

$\left(\frac{1-m_o}{m_o}\right)$ for large m. Thus, the probability of more than one failure during flight before the observance of a successful burst has been included. Hence,

$$T_2 = t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_s$$

$$M_2 = \frac{P_2}{T_2} .$$

Doctrine 3: fire and observe (for homing and/or beam-riding missiles)

$$P_3 = 1.0, \; p > 0$$

$$T_3 = 1\left\{\left[t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_o\right] + t_s\right\} + \left[t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_o\right] \cdot$$

$$\cdot\left[(1-p_o p) + (1-p_o p)^2 + --- + (1-p_o p)^m\right]$$

$$= t_s + \frac{t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_o}{p_o p}, \quad p \geqslant 0$$

$$M_3 = \frac{P_3}{T_3}, \quad \text{all values of p.}$$

This formula bears further explanation in that it represents a change of the thinking pattern and of formula construction. Doctrines 1 and 2 represented no particular difficulties in that there was no question as to how many times a specific time increment entered into the computation. Each increment entered only once in the denominators of the mortality rates except in the case of t_f in doctrine 2 where it was seen that allowance was made for the probability of more than one failure during flight before the observance of a successful burst (in fact, allowance was made for one failure, or two, three, and so on).

In considering doctrine 3, a similar allowance must be made for the probability of more than one observation period

before success (and then add on the switching time increment). But if an allowance of this sort is made, then the numerator of M_3 must also be changed. The fact that a success is not observed during the time t_o does not necessarily mean that a success has not occurred; p_o, the probability of observing a success, may have a value less than 1, and the numerator of M_3 will be successively taking on values larger than p and approaching 1.0 as a limit as additional possible missiles are fired.

These difficulties can be avoided by looking at the mortality rate in the manner presented. The reader may prefer to think of the process as computing the expected time (denominator) for one success (numerator).

It will be noted that if $p=0$, the mortality rate for doctrine 3 becomes zero; if $p=1.0$ (assuming for the moment that $p_o=1.0$), the mortality rate becomes the same as that for doctrine 2 with the addition of the time increment t_o in the denominator; if $p=.5$, the mortality rate for 3 will exceed that for 2 provided t_o is less than t_s --- doctrine 3 here advocates a second shot at the same target, doctrine 2 would fire at a new target; and so on. It is seen after further critical examination that the mortality rate for 3 performs the same desired function as that of 1 and 2 despite the fact that its development was somewhat different.

The next step will be to compare the mortality rates of the several doctrines. Doctrine 3 will be better than doctrine 2 when the following inequality holds:

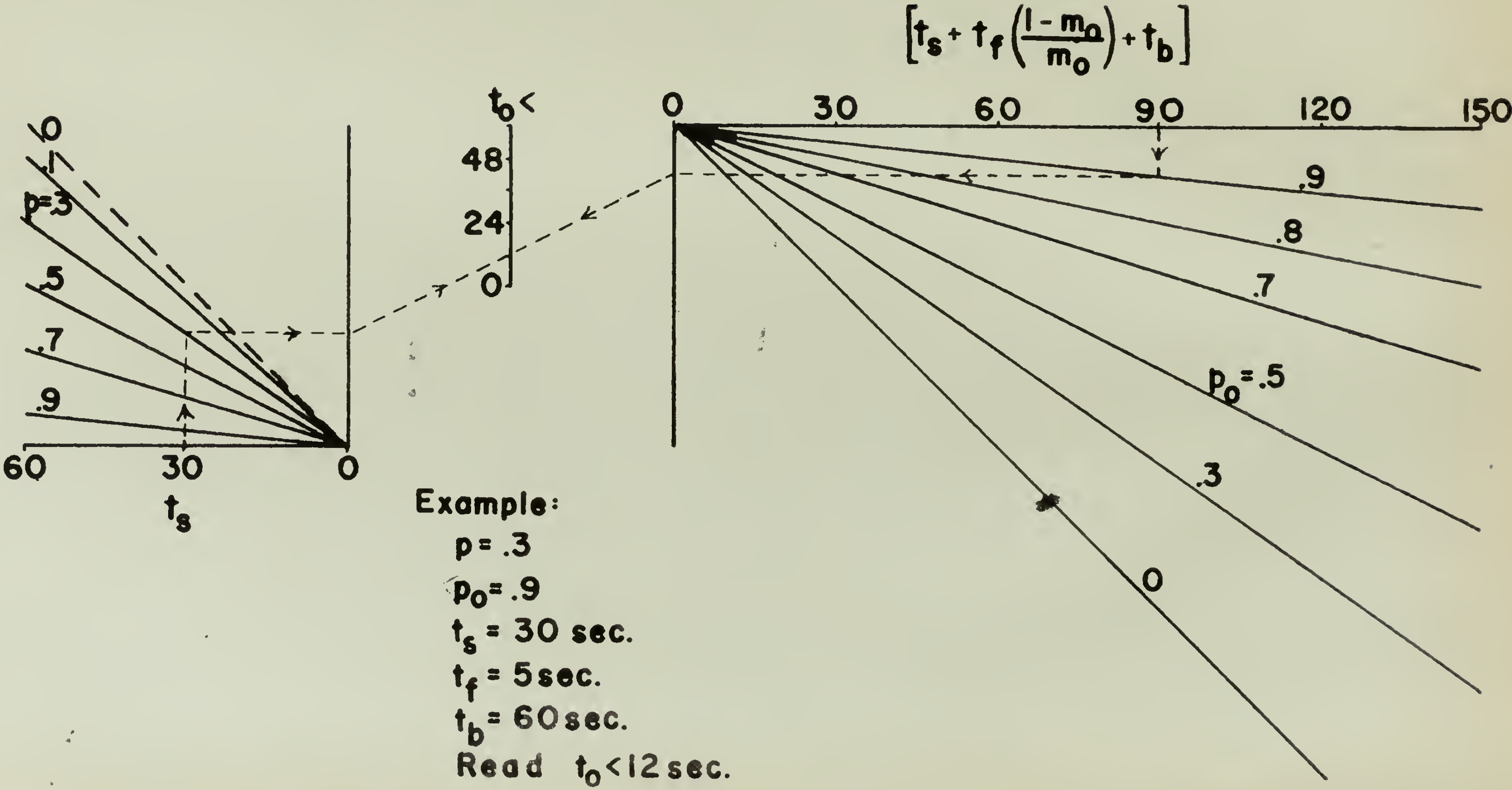
$$M_3 > M_2$$

$$\frac{1}{t_s + \dfrac{t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_o}{p_o p}} > \frac{}{t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_s}$$

or $t_s(1-p)-(1-p_o)\left[t_s + t_f \frac{1-m_o}{m_o} + t_b\right] > t_o > 0$

The equation has been reduced to this particular form so that it may be examined conveniently by a graphic presentation. Figure 6 shows a nomogram of the function developed above. It will be noted that all of the quantities in this function are readily determinable from available information --- t_b as a tabulated function of the firing range, p similarly tabulated on the basis of theoretical considerations concerning the missile itself; m_o, t_s, and t_f from statistical analysis of operations --- except the quantity p_o. p_o is, of course, directly related to t_o itself. If t_o is large, p_o might well be high; that is, the longer the observation time after burst, the more likely that a kill will be observed if it has, in fact, occurred. If but several seconds are available to observe after a burst, there is a smaller probability that a kill will be observed --- in fact, in accordance with the definitions of the four types of kills (A, B, C, and D), "several seconds" implies that only an A kill may be observed during this period (with an attached probability), the possibility of a B kill (with a similar attached probability), and a very small possibility of a C kill (depending on the length of the "several seconds" as compared with the definition "within a few seconds" of the B kill. Increasing t_o thus allows the observer the addition to p_o of an increment to account for the probability

Nomogram of $t_s(1-p) - (1-p_o)\left[t_s + t_f\left(\frac{1-m_a}{m_o}\right) + t_b\right] > t_o > 0$



Example:

$p = .3$

$p_o = .9$

$t_s = 30$ sec.

$t_f = 5$ sec.

$t_b = 60$ sec.

Read $t_o < 12$ sec.

Figure 6

of noting a C kill if it did occur and the target will fall during this early period of the five-minute definition time allowed. (The time scale of these definitions is, of course, a relative one as has been pointed out earlier.) It will be noted that having any $t_o > 0$ would also allow the observer to note missile operability in the firing of a homer since it implies that a second shot is not fired until (at least) a successful burst is observed.

Although the role of the Operations Analyst is traditionally one of impartiality, there is an obligation to present the facts in such a manner as to make the course of decision clear. The author thus feels compelled to introduce a plea for the case of fire and observe. It appears quite evident that when switching time is high and/or kill probability is low, the utilization of an observation period will be beneficial.

Comparing doctrines 3 and 1, it is seen that the former will be better than the latter when the following inequality holds:

$$M_3 > M_1$$

$$\frac{1}{t_s + \dfrac{t_f\left(\frac{1-m_o}{m_o}\right) + t_b + t_o}{p_o p}} > \frac{m_o p}{t_s}$$

or $\frac{1}{m_o}(t_s p_o - t_f) - t_s p_o p + t_f - t_b > t_o > 0$

Again the equation has been reduced to a particular form for graphic presentation purposes. Figure 7 is a nomogram of this function. It is not intended that Figure 7 would have any operational value other than as a preliminary check on

Nomogram of $\frac{1}{m_o}(t_s p_o - t_f) - t_s p_o p + t_f - t_b > t_o > 0$

Example:

$t_s = 25$ sec.

$p_o = 1.0$

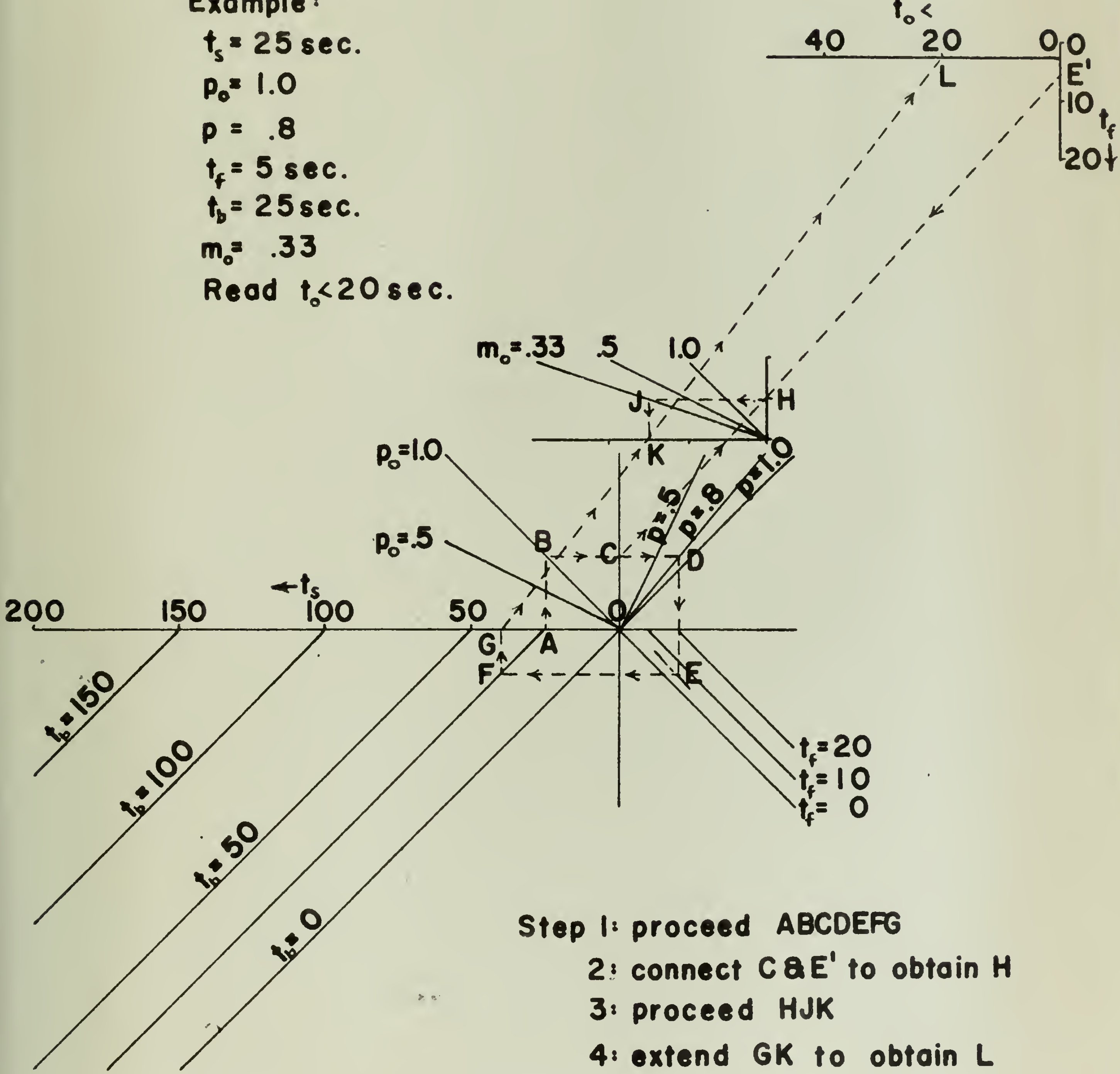
$p = .8$

$t_f = 5$ sec.

$t_b = 25$ sec.

$m_o = .33$

Read $t_o < 20$ sec.



Step 1: proceed ABCDEFG

2: connect C & E' to obtain H

3: proceed HJK

4: extend GK to obtain L

Figure 7

parameter values. Examination of the illustrative example on the figure reveals that all of the assumed values are fairly reasonable except the missile operability (m_o). The extremely low value of m_o=.33 indicates that the use of an observation period of (t_o) 20 seconds is desirable. If a more reasonable value of this parameter had been selected, it will be noted that t_o would be sharply reduced. This latter reduction would in turn reduce p_o (in accordance with the discussion above) which would further reduce t_o. The cumulative effect of this change in m_o would be to produce a negative t_o which is to say that $M_3 > M_1$. Examination of the nomogram for the effect of other possible variations is interesting.

Doctrine 3 could compare favorably with doctrine 1 under conditions of high missile operability when a system which has a high switching time and a low kill probability fires a high-velocity missile (low t_b). Since the effect of employing higher missile velocities will be to decrease the number of possible intercepts on an inbound target, as is shown in Appendix III, increased kill probability and/or increased maximum firing range must accompany such employment (higher missile velocities) for it to be acceptable as an improvement over present systems.

It thus seems doubtful that fire and observe will compare favorably with fire and switch for homers at this time on the basis of the present stage of the art of missilery.

BIBLIOGRAPHY

1. Trueman, R.E. — A THEORETICAL EVALUATION OF AIRCRAFT RAID SIZE REQUIRED TO PENETRATE A MISSILE DEFENSE
Convair Missile Division
August 1956

2. Applied Physics Laboratory, Johns Hopkins University — CLA INTERNAL MEMO 273; DOCTRINE OF FIRE AND OBSERVE (Secret)
CLA Grp APL/JHU
May 1953

3. Applied Physics Laboratory, Johns Hopkins University — CLA INTERNAL MEMO 306; FIRE AND OBSERVE (Secret)
CLA Grp APL/JHU
December 1953

4. Burnham, F.H. — A METHOD FOR COMPARING SURFACE TO AIR GUIDED MISSILE SYSTEMS FOR THE DEFENSE OF NAVAL SURFACE UNITS
U.S.Naval Postgraduate School
1954

5. Tucker, J.F. — SUBMARINE FIRING PHASE DECISIONS
U.S.Naval Postgraduate School
1954

6. National Bureau of Standards — TABLES OF THE BINOMIAL PROBABILITY DISTRIBUTION, APPLIED MATHEMATICS SERIES 6
Nat. Bu. of Std.
1950

7. Walsh, J.E. — INADEQUACY OF COST PER 'KILL' AS MEASURE OF EFFECTIVENESS
Journal of the Operations Research Society of America
December 1957

APPENDIX I

DEVELOPMENT AND ILLUSTRATION OF THE SIMPLE SURVIVAL PROBABILITY MODEL

Where each trial (or missile firing) is assumed an independent event, the development of the simple survival probability model proceeds as follows:

P_k = probability of kill of the aircraft in one shot

P_s = probability of survival of the aircraft

m = number of missiles fired

for $m = 1,\ P_s = 1-P_k$

$m = 2,\ P_s = 1-\left[P_k + (1-P_k)P_k\right] = (1-P_k)^2$

$m = 3,\ P_s = 1-\left[P_k + (1-P_k)P_k + (1-P_k)^2 P_k\right] = (1-P_k)^3$

--- , ---

$m = m,\ P_s = (1-P_k)^m.$

Normally, this paper considers P_k as the probability of kill of an aircraft in one shot and m as the number of shots fired. However, the developments presented are equally valid for salvo fire where several missiles are fired simultaneously. The computational procedure can be accomplished by either of two methods for the simple survival probability model, as follows:

Method 1: let single-shot $P_k = .4$ and consider firing two missiles simultaneously, then

$P_s = (1-.4)^2 = .36$

Method 2: consider <u>one salvo</u> of 2 missiles where the P_k of an individual missile is .4, then

salvo $P_k = \left[.4 + (1-.4).4\right] = .64$, and

$P_s = (1-.64)^1 = .36$, which is the same result

achieved by method 1.

The computation for an inbound target (case (3) of page 5 may also be made by averaging the salvo P_k in which case the close approximation to the P_s is again produced by such averaging process.

APPENDIX II

A FURTHER CONSIDERATION OF THE CUMULATIVE BINOMIAL DISTRIBUTION SURVIVAL PROBABILITY MODEL

As was indicated earlier in this study, the emphasis of the investigation is on application for use by a firing doctrine planner. A considerable variation of the parameters P_k and N_s (introduced in Chapter I) at the same time would probably only occur as a result of a change in design of the system itself, hence a complete investigation of such variation is not contemplated here. But it is interesting to observe that the change-over point for use of the proposed doctrine B instead of doctrine A occurs at N=4, which turns out to be the saturation level of doctrine A. Surely, this is not a coincidence! As a matter of fact, it takes but a moment's reflection and a short analysis to detail the conditions under which change-over will always occur at the stated point. Since the planner is working with a given installation (and may only vary his parameters by employment of devices such as firing two missiles per salvo, rearranging the spacing of the ships in the formation (or batteries in a shore installation), and so on), his criterion is sufficiently determined by the analysis previously presented. The designer, however, might well investigate further this particular facet of the problem.

Examining Figure 1, it is seen that, under certain conditions, doctrine A (in the example presented) might again be the more desirable doctrine --- in particular, when

$$(N-N_{s_B}+n_B) - (N-N_{s_A}+n_A) > 0.$$

The problem then becomes one of designing a system around a given saturation level and salvo kill probability. The author feels, however, that this insert might well be of more interest to the mathematician than to the practical designer.

It is of additional interest to observe that the measure developed could be used by friendly forces in the determination of the number of aircraft necessary to penetrate an enemy defense. P_k should not be too difficult to estimate; N_s could assume any value in its possible range $0 \leqslant N_s \leqslant \infty$, the higher values being used with conservatism.

An additional very broad application of the measure comes to mind. Assume for the moment that White and Red are at war and that White can attack Red with aircraft (or ship or submarine-launched, long-range missiles) from the four points of the compass, from each of these directions with a different probability of penetration depending on the weather in that location, enemy strength, the location of suitable Red targets, and so on. White's mission is to destroy (by special weapons) x% of the Red targets with N aircraft available to White. The tactical planner's problem is to determine the optimum deployment of the aircraft (or ships, etc.) for accomplishing the mission.

Knowing the capabilities of the particular weapon to be carried and the number carried by each plane, it is seen that the number of planes that must actually release bombs to accomplish the factor x% may readily be determined. This latter number will be equivalent to n in the development of Chapter I. By suitable analysis and approximation, one should be able to

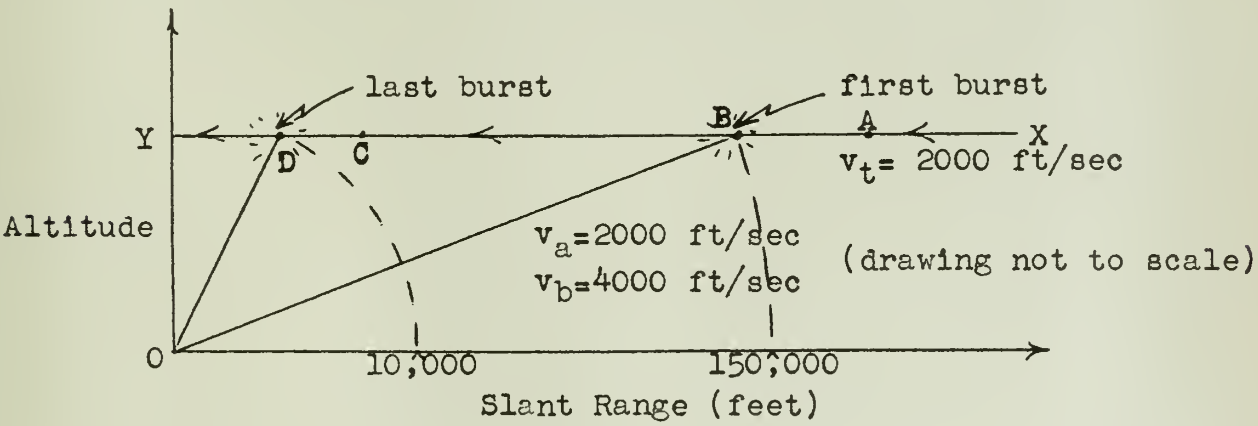
determine P_k values for the several directions involved, at least order-of-magnitude estimates and certainly in the proper order (that is, north might be the most difficult direction of attack and have a P_k of .7, south might be the easiest with P_k=.2, and so on). Everything is now determined in accordance with the Chapter I development except the quantity P_n. This latter factor can be used as the measure of effectiveness by the tactical planner in his determination of the number of planes to attack Red from each direction (the sum of which equals N). The arrangement with the highest P_n offers the most effective attack since it accomplishes the required mission with the highest (survival) probability (and therefore has a higher probability of doing even more damage than the x%). Changes in the required damage (which could result from changes in the mission)(x%); the number of aircraft available (N); and the enemy strength, weather, etc. (P_k) will reflect in this value of P_n. Game theory techniques could be used to determine accurately the highest value of P_n.

APPENDIX III

A CONSIDERATION OF TARGET INTERCEPTS VERSUS MISSILE VELOCITY

Although the matter of the number of target intercepts available at a given missile velocity is slightly removed from the intent of this paper, the statement has been made (page 41) that increased missile velocity will result in fewer intercepts and this unusual conclusion bears proof.

Consider the following missiles where v_1 is given as an average velocity over the total path, for convenience the range R is slant range (with a maximum firing range of 150,000 feet and a minimum of 10,000 feet) and the target is inbound to the launching site with a constant velocity of 2000 ft/sec. It is assumed that increased missile velocity has <u>not</u> increased the maximum or minimum firing ranges.



Target is inbound on path XY. In order to have the first burst occur at the maximum range, the first missile must be launched when the target is at position A. OB (in seconds)

$= \frac{150000}{2000} = 75$ seconds; hence AB (in seconds) $= 75$ seconds.

OD $= \frac{10000}{2000} = 5$ seconds; hence CD $= 5$ seconds in order for last

burst to occur at point of minimum firing range. BD = $\frac{150000-10000}{2000}$ = 70 seconds. Therefore AC = AB + (BD-CD) = 75 + (70-5) = 140 seconds= t_a = total time available for firing missiles with velocity v_a.

Replacing missile "a" with missile "b", v_b= 4000 ft/sec:

$$OB = \frac{150000}{4000} = 37.5 \text{ seconds} = AB$$

$$OD = \frac{10000}{4000} = 2.5 \text{ seconds} = CD$$

$$BD = \frac{150000-10000}{2000} = 70 \text{ seconds}$$

AC = 37.5 + (70-2.5) = 105 seconds = t_b = total time available for firing missiles with velocity v_b.

It is clear that, since $t_a > t_b$, more intercepts can be achieved using missile "a" than missile "b", the firing rate remaining the same for both missiles. Thus, it is seen that increased missile velocity must be accompanied by increased kill probability and/or increased maximum firing range for it to be acceptable as an improvement over present systems.

E — expected kills

E_a — maximum single-shot expectation of kill (given that the target is "within the envelope" and there is a "missile available") per plane times the number of planes

M.E. — measure of effectiveness

M_i — expected mortality rate (kills/second) for an i^{th} doctrine, $= P_i/T_i$

N — number of attacking aircraft; raid size

N_s — saturation level or the maximum number of aircraft that can be engaged at any one instant using a particular doctrine

P — eventual state of missile, where $P=1.0$ for success (kill obtained) and $P=0.0$ for failure (kill not obtained)

P_i — expected engagement kill probability for an i^{th} doctrine

P_k — probability of kill of an aircraft in one shot

P_n — probability that n or more aircraft out of a total N aircraft will survive a given defense

P_s — (simple) probability of survival of an aircraft

T_i — expected total engagement time for an i^{th} doctrine

g_o — ground system operability or the number of times the ground system operates in a manner sufficient to launch a missile divided by the total number of attempts to operate the ground system

g_r — ground system reliability or that portion of the number of times when the ground system did operate that it was also within proper tolerances for a successful missile launch

m — number of missiles fired

m_o — missile operability or the number of times the component parts of the missile operate in such a manner as to cause a proper burst divided by total attempts

m_r — missile reliability or that portion of the number of times when the missile did operate that the component parts were also within proper time tolerances for a successful burst

n	the minimum number of aircraft that will survive a given defense in accordance with the laws of probability with the total number of attacking aircraft is N ($N > n$) and the probability of kill and survival probability are P_k and P_n respectively
p	kill probability of an operable missile (which is a function of miss distance, the latter being a function of ground system reliability and missile reliability)
p_o	probability of observing a kill within time t_o
t_b	time from launch to burst
t_f	missile mean failure time (where failure occurs after launch and before burst)
t_l	time to reload launcher
t_o	average time after burst for observing a kill
t_s	time required to switch to a different target (measured from "leave the present target" to "fire at a different target")
v	average missile velocity over the total path from launch point to point of burst
$x\%$	percentage of enemy targets to be destroyed in accordance with a given mission